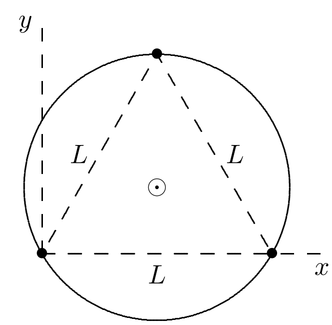
55. Each star is attracted toward each of the other two by a force of magnitude GM^2/L^2, along the line that joins the stars. The net force on each star has magnitude $2(GM^2/L^2)\cos 30°$ and is directed toward the center of the triangle. This is a centripetal force and keeps the stars on the same circular orbit if their speeds are appropriate. If R is the radius of the orbit, Newton's second law yields $(GM^2/L^2)\cos 30° = Mv^2/R$.

The stars rotate about their center of mass (marked by $\odot$ on the diagram to the right) at the intersection of the perpendicular bisectors of the triangle sides, and the radius of the orbit is the distance from a star to the center of mass of the three-star system. We take the coordinate system to be as shown in the diagram, with its origin at the left-most star. The altitude of an equilateral triangle is $(\sqrt{3}/2)L$, so the stars are located at $x = 0$, $y = 0$; $x = L$, $y = 0$; and $x = L/2$, $y = \sqrt{3}L/2$. The x coordinate of the center of mass is $x_c = (L + L/2)/3 = L/2$ and the y coordinate is $y_c = (\sqrt{3}L/2)/3 = L/2\sqrt{3}$. The distance from a star to the center of mass is $R = \sqrt{x_c^2 + y_c^2} = \sqrt{(L^2/4) + (L^2/12)} = L/\sqrt{3}$.



Once the substitution for R is made Newton's second law becomes $(2GM^2/L^2)\cos 30° = \sqrt{3}Mv^2/L$. This can be simplified somewhat by recognizing that $\cos 30° = \sqrt{3}/2$, and we divide the equation by M. Then, $GM/L^2 = v^2/L$ and $v = \sqrt{GM/L}$.